AERO 452: Spaceflight Dynamics II
HW 2

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1 Handwork

2 MATLAB Results

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1: Work by hand

AERO452 HW#2

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1. At time t=0 a particle is at the origin of a CW frame with a relative velocity of [1 -1 1] (m/s). What will be the relative speed of the particle to the origin after 1/4th of a period?

USE CW EQNS.

$$\delta\dot{x}(t) = \delta\dot{x}_0 \cos(nt) + (3n\,\delta x_0 + 2\,\delta\dot{y}_0)\sin(nt)$$

$$\delta\dot{y}(t) = (6n\,\delta x_0 + 4\,\delta\dot{y}_0)\cos(nt) - 2\,\delta\dot{x}_0 \sin(nt) \cdots$$

$$- (6n\,\delta x_0 + 3\,\delta\dot{y}_0)$$

$$\delta\dot{z}(t) = -\delta z_0\, n \sin(nt) + \delta\dot{z}_0 \cos(nt)$$

SINCE AT $t=0$ THE PARTICLE WAS AT THE ORIGIN OF A CW FRAME,

$$\delta x_0 = \delta y_0 = \delta z_0 = 0 \text{ m/s}$$

FURTHERMORE, WE ARE GIVEN:

$$\delta\dot{x}_0 = 1$$
$$\delta\dot{y}_0 = -1$$
$$\delta\dot{z}_0 = 1$$

KNOWNS

KNOWING THAT $n \equiv$ MEAN MOTION

$$= \frac{2\pi}{T}$$

AND THUS $t = \frac{1}{4}P$

INPUTS TO THE CW EQNS.

WE USE MATLAB AND THE MATRIX METHOD TO FIND

$\delta\dot{x}, \delta\dot{y}, \delta\dot{z}$.

SEE

MATLAB

OUTPUT :

```
Command Window
~~~~~~~~~~~~~~ PROBLEM 1 ~~~~~~~~~~~~~~
Initial speed is: 1.7321 m/s
New dv is:

dv =

    -2
     1
     0

Speed is: 2.2361 m/s
fx HEART CHECK: speed is not unreasonable.
```

$$\delta v = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \text{m/s} \longleftarrow$$

SPEED: 2.236 m/s $\longleftarrow$

2. Curtis 7.15

7.15 A GEO satellite strikes some orbiting debris and is found 2 h afterward to have drifted to the position $\delta\mathbf{r} = -10\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$(km) relative to its original location. At that time the only slightly damaged satellite initiates a two-impulse maneuver to return to its original location in 6 h. Find the total delta-v for this maneuver.
{Ans.: 3.5 m/s}

ASSUMPTIONS: · CIRCULAR ORBIT (TARGET).

· CLOSE ENOUGH PROXIMITY S.T. THE CW EQNS. HOLD. (FEW KM'S)

KNOWN. GEO ∴ ALT = 35,786 km.
ORBIT = CIRCULAR

AT $t = 2$ HOURS
$= 2(3600)$ sec
$= 7200$ sec ←

$\delta r = [-10, 10, 0]$ km

FIRST ORDER OF BUSINESS: FIND CURRENT VELOCITY (VECTOR FORM).

FOR GEO, CIRCULAR, $T = 24$ H
$= 86400$ sec. ←

$$n = \frac{2\pi}{T}$$

$$t = 2H = \frac{1}{12}T.$$

$$\therefore\ nt = \frac{2\pi}{T} \cdot \frac{1}{12}T$$

$$= \frac{\pi}{6} \leftarrow$$

USE CW EQNS, KNOWING THAT: $\delta \dot{x}_0 = \delta \dot{y}_0 = \delta \dot{z}_0 = 0$

(IT WAS AT THE CENTER
OF A CW FRAME)

$\boxed{\text{CW EQNS}}$

FOR $\delta x = -10\text{ km}$
$\delta y = 10\text{ km}$
$\delta t = 0$

$t = \frac{1}{12}T$ OR 7200sec,

WE HAVE:

$$\delta x(t) = 4\cancelto{0}{\delta x_0} + \frac{2\delta \dot{y}_0}{n} + \frac{\delta \dot{x}_0}{n}\sin(nt)\ldots \qquad (1.A)$$

$$-\left(3\cancelto{0}{\delta x_0} + \frac{2\delta \dot{y}_0}{n}\right)\cos(nt)$$

$$\delta y(t) = 2\frac{\delta \dot{x}_0}{n}\cos(nt) + \left(6\cancelto{0}{\delta x_0} + \frac{4\delta \dot{y}_0}{n}\right)\sin(nt)\ldots \qquad (1.B)$$

$$-\left(6n\cancelto{0}{\delta x_0} + 3\delta \dot{y}_0\right)t - \frac{2\delta \dot{x}_0}{n} + \cancelto{0}{\delta y_0}$$

$$\delta z(t) = \cancelto{0}{\delta z_0 \cos(nt)} + \frac{\delta \dot{z}_0}{n}\sin(nt) \qquad (1.C)$$

SOLVING THE z-TERM GIVES:

$$0 = \frac{\delta \dot{z}_0}{n}\sin(nt)$$

$$\delta \dot{z}_0 = 0 \longleftarrow$$

SOLVING THE x- AND y- EQUATIONS SIMULTANEOUSLY GIVES:

$$\begin{array}{c|ccc} & \delta \dot{x}_0 & \delta \dot{y}_0 & \text{CONSTANT} \\ x & n^{-1}\sin(nt) & \frac{2}{n} - \frac{2}{n}\cos(nt) & 10 \\ y & \frac{2}{n}\cos(nt) - \frac{2}{n} & \frac{4}{n}\sin(nt) - 3t & -10 \end{array}$$

USING RREF WE HAVE THAT:

$$\delta v_0 = [\,0.0014 \quad -0.0014 \quad 0\,)\ \text{km/sec}$$

WE NOW HAVE $\underbrace{\delta r_0,\ \delta v_0}_{\text{AT IMPACT LOCATION.}},\ \underbrace{\delta r_1,\ \delta v_1}_{\text{START OF PROBLEM}}$

RECALL THE CW EQNS. FOR TWO IMPULSE BURNS:

FINAL VELOCITY (SECOND BURN)

$$\delta \vec{v}_0^{\,+} = \left[\Phi_{rr}(t_f)\right]^{-1}\left[-\Phi_{rr}(t_f)\right]\left\{\delta \vec{r}_0\right\} \qquad (1)$$

INITIAL VELOCITY (FIRST BURN)

$$\delta v_0^{-} = [\,0.0014 \quad -0.0014 \quad 0\,)\ \text{km/sec}$$

SEE MATLAB FOR WORK.

FINAL OUTPUT:

```
~~~~~~~~~~~~~~ PROBLEM 2 ~~~~~~~~~~~~~~
TOTAL DELTA-V for this maneuver is:
3.4883 [m/s]
HEART CHECK: seems reasonable considering the other speeds involved.
```

♡

3. A target is in a 300 km circular geocentric orbit. The chaser is [-1;0;0] km behind the target with the same velocity when it initiates a two-impulse maneuver in an effort to rendezvous at the target within 30 mins.

a) Find the total delta-v.

BY PROBLEM PARAMETERS, CW EQNS. WILL HOLD.

$\delta r = [-1, 0, 0]$ km

$\delta v = [0, 0, 0]$ m/s (SAME VELOCITY)

$t = 30$ min.

ALTITUDE = 300 km

FIND TOTAL δv.

WE USE THE MATRIX FORM OF THE CW EQNS TO SOLVE FOR TOTAL Δv.

SEE MATLAB.

MATLAB

FINAL OUTPUT: 3.A)

```
~~~~~~~~~~~~~~ PROBLEM 3 ~~~~~~~~~~~~~~
Problem 3.a: Delta-V Total is: 2.8575 m/s
fx >>
```

♡ SEEMS REASONABLE.

4. Spacecraft A and B are in coplanar, circular geocentric orbits. The orbital radii are 8000 km for the target spacecraft, A, and 7000 km for the chaser spacecraft, B. Assume the LVLH frame is with x radially outward and y is in the direction of the velocity of the target. When B is directly below A at perigee, answer these three questions:

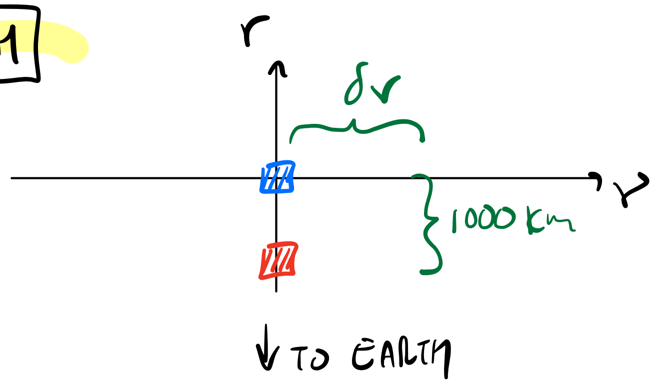
a) Knowing what you do about the trend analysis, describe the direction of the relative motion.

b) Without using the CW equations and the close proximity equation (meaning use the actual relative motion equations), calculate the relative speed of B to A?

KNOWNS:

	TARGET	CHASER
r	8000 km	7000 km
ORBIT:	COPLANAR, CIRCULAR, GEOCENTRIC	

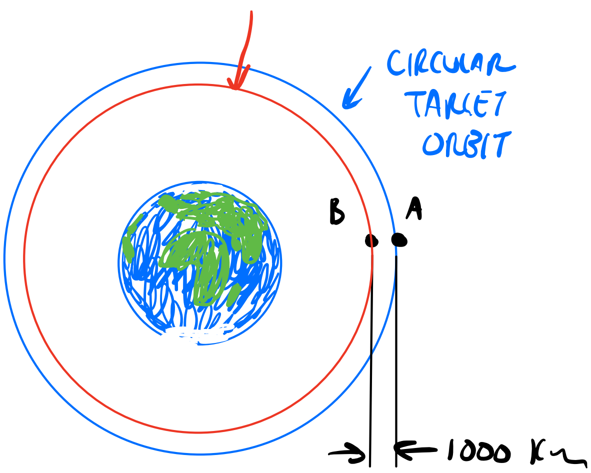
LVLH



CHASER AT PERIGEE

TARGET CIRCULAR

IN ECI



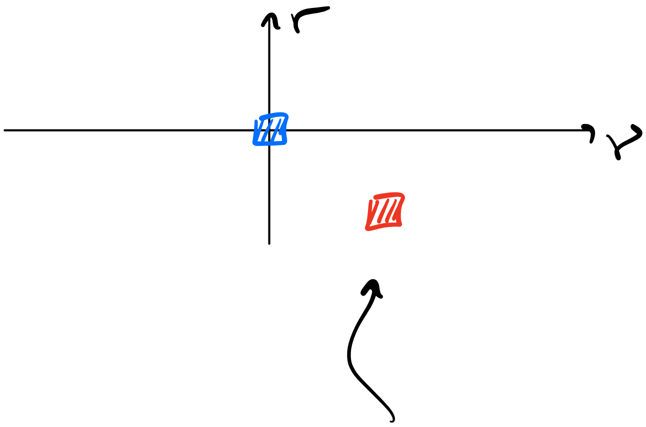
a) Knowing what you do about the trend analysis, describe the direction of the relative motion.

$$v_{circ} = \sqrt{\frac{\mu}{r}}$$

$v_A = 7.0587$ km/sec

$v_B = 7.5460$ km/sec

IN THE LVLH FRAME, THIS LOOKS LIKE:



- SINCE ORBITS ARE BOTH COPLANAR AND CIRCULAR (CONCENTRIC)

$$\delta r = \begin{bmatrix} -1000 \\ 0 \\ 0 \end{bmatrix} \text{ km}$$

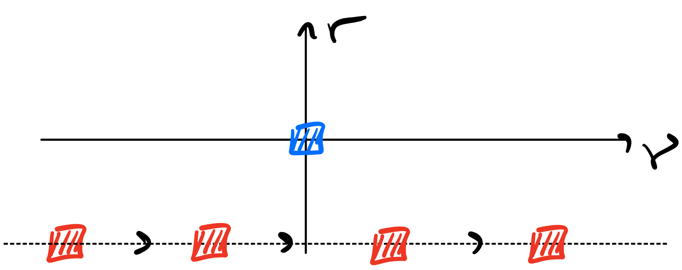
AT PERIGEE

$$\delta v = \begin{bmatrix} 0 \\ 0.4874 \\ 0 \end{bmatrix} \text{ km/sec}$$

- CHASER IS FASTER.
- BUT ALTITUDE DOES NOT CHANGE.

DIRECTION OF RELATIVE MOTION

SINCE THE CHASER IS MOVING FASTER THAN THE TARGET, IN THE RELATIVE FRAME THE CHASER WILL APPEAR TO MOVE AWAY ($+\delta v$).



- CONSTANT ALTITUDE (r)
- CONSTANT VELOCITY (v)
- CHASER PASSES TARGET BY

TREND ANALYSIS PART (IN LIGHT OF THE LECTURE!)

- SINCE ALTITUDE DIFFERENCE (1000 km) IS FIXED, THE RELATIVE MOTION IS IN THE y-DIRECTION.

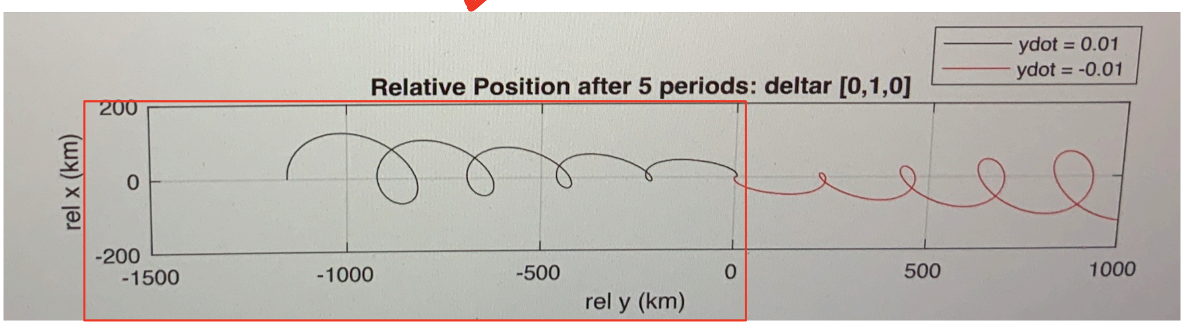
NAMELY,

i) ANY CHANGE IN $\dot{y}$ PRODUCES ELLIPTICAL MOTION.

ii.) CHANGE IS SEEN IN ECCENTRICITY AND SEMI-MAJOR AXIS (AND ∴ PERIOD).

iii) INCREASE IN VELOCITY (SINCE CHASER IS LOWER; FASTER AFTER PERIGEE).

WE EXPECT THE PLOT OF RELATIVE MOTION TO LOOK LIKE THE LEFT SIDE



b) Without using the CW equations and the close proximity equation (meaning use the actual relative motion equations), calculate the relative speed of B to A?

IS IT CLOSE PROXIMITY?

$\delta r = 1000\,km$.

→ NO.

∴ USE EQUATIONS OF MOTION AND SOLVE FOR RELATIVE (LVLH) VELOCITY.

RELATIVE MOTION EQNS. → SEE MATLAB
(FROM WEEK 1)

$$\boxed{\text{RELATIVE SPEED} = v_{rel_x} = 1.3697 \text{ km/sec}} \leftarrow$$

♡ SEEMS REASONABLE. FEELS A LITTLE FAST, BUT THE LVLH FRAME IS ROTATING.

END HW #2

2: MATLAB RESULTS

Table of Contents

Problem 1

```
~~~~~~~~~~~~~ PROBLEM 1 ~~~~~~~~~~~~~
Initial speed is: 1.7321 m/s
New dv is:

dv =

    -2
     1
     0

Speed is: 2.2361 m/s
HEART CHECK: speed is not unreasonable.
```

Problem #2: Curtis 7.15

```
~~~~~~~~~~~~~ PROBLEM 2 ~~~~~~~~~~~~~
TOTAL DELTA-V for this maneuver is:
3.4883 [m/s]
HEART CHECK: seems reasonable considering the other speeds involved.
```

PROBLEM #3.

```
~~~~~~~~~~~~~ PROBLEM 3 ~~~~~~~~~~~~~
Problem 3.a: Delta-V Total is: 2.8575 m/s
```

PROBLEM #4.

```
~~~~~~~~~~~~~ PROBLEM 4 ~~~~~~~~~~~~~
4.a: See handwork.

4.b: Relative Motion (not close proximity)
Relative speed at perigee is: 0.48737 km/s
Relative speed is: 1.3697 km/s
```

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3: MATLAB SCRIPT

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```
%{
AERO 452: Spaceflight Dynamics II
HW #2
Author: Justin Self
Cal Poly
%}

% Housekeeping
clear all; close all; clc;
addpath("C:\MATLAB_CODE\Orbits\")

% Constants
mu = 398600; % km3/s2
r_earth = 6378; % km
```

Problem 1

```
%{
At t = 0 a particle is at the origin of a CW frame with a relative velocity
of [1 -1 1] m/s
FIND the relative speed of the particle to the origin after 1/4 period
%}

% Use CW equations to find dv

syms P n
state0 = [0;0;0;1;-1;1];
period = P;
t = P/4;

initial_speed = norm([1;-1;1]);

% Print results Prob 1
disp("~~~~~~~~~~~~~ PROBLEM 1 ~~~~~~~~~~~~~")
disp("Initial speed is: " + initial_speed + " m/s")

[dr,dv] = clohessy_wiltshire_eqs(state0,period,t);
dv = double(dv);
speed = norm(dv);
disp("New dv is: ")
dv
disp("Speed is: " + speed + " m/s")
```

```
disp("HEART CHECK: speed is not unreasonable.")
disp(" ")
```

Problem #2: Curtis 7.15

```
disp("~~~~~~~~~~~~~~ PROBLEM 2 ~~~~~~~~~~~~~~")
% Assumptions:
% Circular orbit
% velocity is constant due to instantaneous OD impact

% Knowns
alt = 35786; % km for GEO, typical
r = r_earth + alt; % km
T = 24*3600; % seconds; GEO period
n = 2*pi/T; % mean motion, s^-1
t = 2*3600; % seconds; this is two hours of drifting from the origin point

% Initial position = [0 0 0]
% Start of problem position (POSITION 1)
dr = 1000.*[-10;10;0]; % in m now

% Find current velocity vector using CW equations
dv0_MINUS = CW_reverse(n,t,dr);

% NOTE that position 1 corresponds to t = 2 hours (7200 sec).

% Ready for CW function call and PRINT
t = 6*3600; % return maneuver in 6 hours
[dv0_PLUS_start_burn,dvf_MINUS_off_burn] = cw_twoimpulse(dr,T,t);

% % dv0- and dv0+ are both known. Thus, deltav to get ON the traj is:
deltav0 = dv0_PLUS_start_burn - dv0_MINUS;

% Report final delta V
deltavf = -dvf_MINUS_off_burn;
DELTAV_TOTAL = norm(deltav0) + norm(deltavf);

disp("TOTAL DELTA-V for this maneuver is: ")
disp(DELTAV_TOTAL + " [m/s]")
disp("HEART CHECK: seems reasonable considering the other speeds involved.")
```

PROBLEM #3.

```
disp(" ")
disp("~~~~~~~~~~~~~~ PROBLEM 3 ~~~~~~~~~~~~~~")

% Part a. Find total delta v for two-impulse maneuver for 30 minute
% rendezvous with target

% Knowns
dr = 1000.*[-1;0;0]; % in m
dv = [0;0;0]; % in m/s (same as target)
```

```
alt = 300; % km
r = r_earth + alt; % in km
t = 30*60; % 30 min in seconds.

% Find orbital period
T = 2*pi*sqrt(r^3/mu);

% Find delta V.
[dv0_PLUS_start_burn,dvf_MINUS_off_burn] = cw_twoimpulse(dr,T,t);

% Get on traj burn
deltav0 = dv0_PLUS_start_burn - dv;

% Get off traj burn
deltavf = -dvf_MINUS_off_burn;

% Total delta-v for this maneuver
DELTAV_TOTAL = norm(deltav0) + norm(deltavf);

% Print results
disp("Problem 3.a: Delta-V Total is: " + DELTAV_TOTAL +  " m/s")
```

PROBLEM #4.

```
disp(" ")
disp("~~~~~~~~~~~~~~ PROBLEM 4 ~~~~~~~~~~~~~")
clear variables

% constants needed
mu = 398600;
r_earth = 6378;

% Knowns
r.a = 8000; % target, in km CIRCULAR
r.b = 7000; % chaser, in km CIRCULAR

% Orbital periods
T.a = 2*pi*sqrt(r.a^3/mu);
T.b = 2*pi*sqrt(r.b^3/mu);

% circular orbit velocities
v.a = sqrt(mu/r.a);
v.b = sqrt(mu/r.b);

disp("4.a: See handwork.")
disp(" ")
disp("4.b: Relative Motion (not close proximity)")

% Create r,v vectors based on knowns.
% IN LVLH
h.a = findh(r.a,mu,0,0);
rVect.a = [8000;0;0]; % km
rVect.b = [7000; 0; 0]; % km
```

```
vVect.a = [0;v.a;0]; % km/s
vVect. b = [0;v.b;0]; % km/s

rel_speed_perigee = v.b - v.a;
disp("Relative speed at perigee is: " + rel_speed_perigee + " km/s")

% With r and v known (in ECI) we may proceed.
[r_relx, v_relx, a_relx] = rva_relative(rVect.a,vVect.a,rVect.b,vVect.b);
disp("Relative speed is: " + norm(v_relx) + " km/s")
```

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